11

Human vision

The recipient of the light emitted by most visible LEDs is the human eye. In this chapter the properties of the human eye are summarized, in particular the properties relating to the light intensity and color as perceived by the human eye.

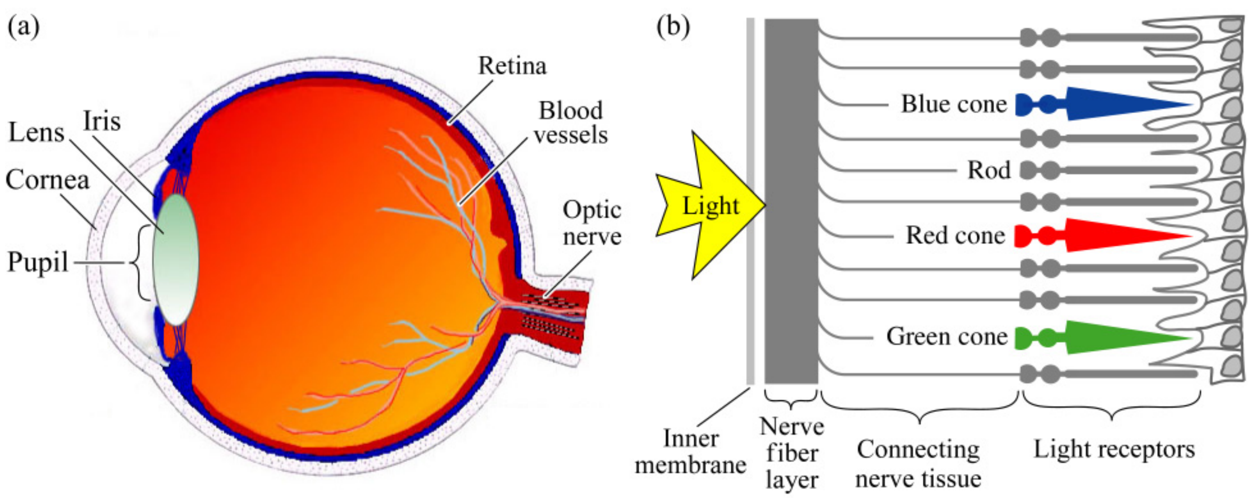


Fig. 11.1. (a) Cross section through human eye. (b) Schematic view of retina including rod and cone light receptors (adapted from Encyclopedia Britannica, 2001).

Light receptors of the human eye

Figure 11.1 shows a schematic illustration of the human eye (Encyclopedia Britannica, 2001). The inside of the eyeball is clad by the retina which is the light-sensitive part of the eye. The inset of the figure shows the cell structure of the retina including the light-sensitive ***rod cells*** and ***cone cells*** and the nerves transmitting the information to the brain. The rod cells are more abundant and more light sensitive than the cone cells. The rods are sensitive in the entire visible spectrum. There are three types of cone cells, namely cone cells sensitive in the red, green, and blue spectral range. The cone cells are therefore denoted as the red, green, and blue cones.

Photopic vision relates to human vision at high ambient light levels (*e. g.* during daylight conditions) when vision is mediated by the cones. ***Scotopic vision*** relates to human vision at low ambient light levels (*e. g.* at night) when vision is mediated by the rods. The rods have a much higher sensitivity than the cones. The sense of colors is essentially lost in the scotopic vision regime. At night, objects lose their colors and only appear as objects with different gray levels. The following discussion relates to the photopic vision regime.

Basic radiometric and photometric units

The physical properties of electromagnetic radiation are characterized by ***radiometric units***. Using radiometric units, we can characterize light in terms of physical quantities, for example the number of photons, photon energy, and ***optical power*** (in the lighting community frequently called the ***radiant flux***). However, the radiometric units are irrelevant when it comes to light perception by a human being. For example, infrared radiation causes no luminous sensation in the eye. To characterize the light and color sensation by the human eye, ***photometric units*** are used.

The ***luminous intensity***, which is a photometric quantity, represents the light intensity of an optical source, as perceived by the human eye. The luminous intensity is measured in units of ***candela*** (cd) which is a unit of the International System of Units (SI unit). The present definition of luminous intensity is as follows: *A monochromatic light source emitting an optical power of (1 / 683) Watt at 555 nm into the solid angle of one steradian (sr) has a luminous intensity of one candela* (1 cd).

The unit of *candela* also has historical significance. All light intensity measurements can be traced back to the candela. It evolved from an older unit, the ***candlepower***, or simply, the ***candle***. One candlepower had been defined as the light intensity emitted from a real candle, similar to a plumber's candle. This candle had specified construction and dimensions.

One standardized candle emits a luminous intensity of about 1.0 cd

The luminous intensity of a light source can thus be characterized by giving the number of standardized candles that together emit the same luminous intensity. Note that *candlepower* and *candle* are non-SI units that are rarely used at the present time.

The ***luminous flux***, which is also a photometric quantity, represents the light power of a source as perceived by the human eye. The unit of luminous flux is the ***lumen*** (lm). It is defined as follows: *A monochromatic light source emitting an optical power of (1 / 683) Watt at 555 nm has a luminous flux of one lumen* (1 lm). The lumen is an SI unit.

Comparison of the definitions for the candela and lumen reveals that one candela equals one lumen per steradian or cd = lm / sr. Thus, an isotropically emitting light source with luminous intensity of 1 cd has a luminous flux of 4π lm = 12.57 lm.

The ***illuminance*** is the luminous flux incident per unit area. The illuminance measured in ***lux*** (lux = lm / m^2). It is an SI unit used when characterizing the illumination in certain environments. ***Table*** 11.1 gives typical values of the illuminance in different environments.

Illumination condition	***Illuminance***
Full moon	1 lux
Street lighting	10 lux
Home lighting	30 to 300 lux
Office desk lighting	100 to 1,000 lux
Surgery lighting	10,000 lux
Direct sunlight	100,000 lux

Table 11.1. Typical illuminances in different environments.

Exercise. ***Photometric units***. A 60 W incandescent light bulb has a luminous flux of 1,000 lm. Assume that light is emitted isotropically from the bulb.

(a) What is the luminous efficiency (*i. e.* number of lumens emitted per Watt of electrical input power) of the light bulb?
(b) What number of standardized candles that emit the same luminous intensity?
(c) What is the illuminance, E_{lum}, in units of lux, on a desk located 1.5 m below the bulb?
(d) Is the illuminance level obtained under (c) sufficiently high for reading?
(e) What is the luminous intensity, I_{lum}, in units of candela, of the light bulb?
(f) Derive the relationship between the illuminance at a distance r from the light bulb, measured in *lux*, and the luminous intensity, measured in *candela*?
(g) Derive the relationship between the illuminance at a distance r from the light bulb, measured in *lux*, and the luminous flux, measured in *lumen*?
(h) The definition of the cd involves the optical power of (1 / 683) W. What, do you suppose, is the origin of this particular power level?

Solution: (a) 16.7 lm / W (b) 80 candles (c) E_{lum} = 35.4 lm / m^2 = 35.4 lux (d) Yes (e) 79.6 lm / steradian = 79.6 cd (f) $E_{\text{lum}}\, r^2 = I_{\text{lum}}$ (g) $E_{\text{lum}}\, 4\pi r^2 = \Phi_{\text{lum}}$. (h) Originally, the unit of luminous intensity had been defined as the intensity emitted by a real candle. Subsequently the unit was defined as the intensity of a light source with specified wavelength and optical power. When the power of that light source is (1 / 683) W, it has the same intensity as the candle. Thus this particular power level has historical reasons and results from the effort to maintain continuity.

$$\ln [V(\lambda)] = A_0 + A_1 \lambda^1 + A_2 \lambda^2 + A_3 \lambda^3 + A_4 \lambda^4 + A_5 \lambda^5$$

380 nm $\leq \lambda \leq$ 520 nm: ($R^2 = 0.9988$)

$A_0 = -502.77,\ A_1 = -1.3992,\ A_2 = 0.0336,\ A_3 = -0.0001,\ A_4 = 2.0 \times 10^{-7},\ A_5 = -1.0 \times 10^{-10}$

520 nm $\leq \lambda \leq$ 750 nm: ($R^2 = 0.9998$)

$A_0 = 1072.6,\ A_1 = -8.7481,\ A_2 = 0.0276,\ A_3 = -4.0 \times 10^{-5},\ A_4 = 3.0 \times 10^{-8},\ A_5 = -9.0 \times 10^{-12}$

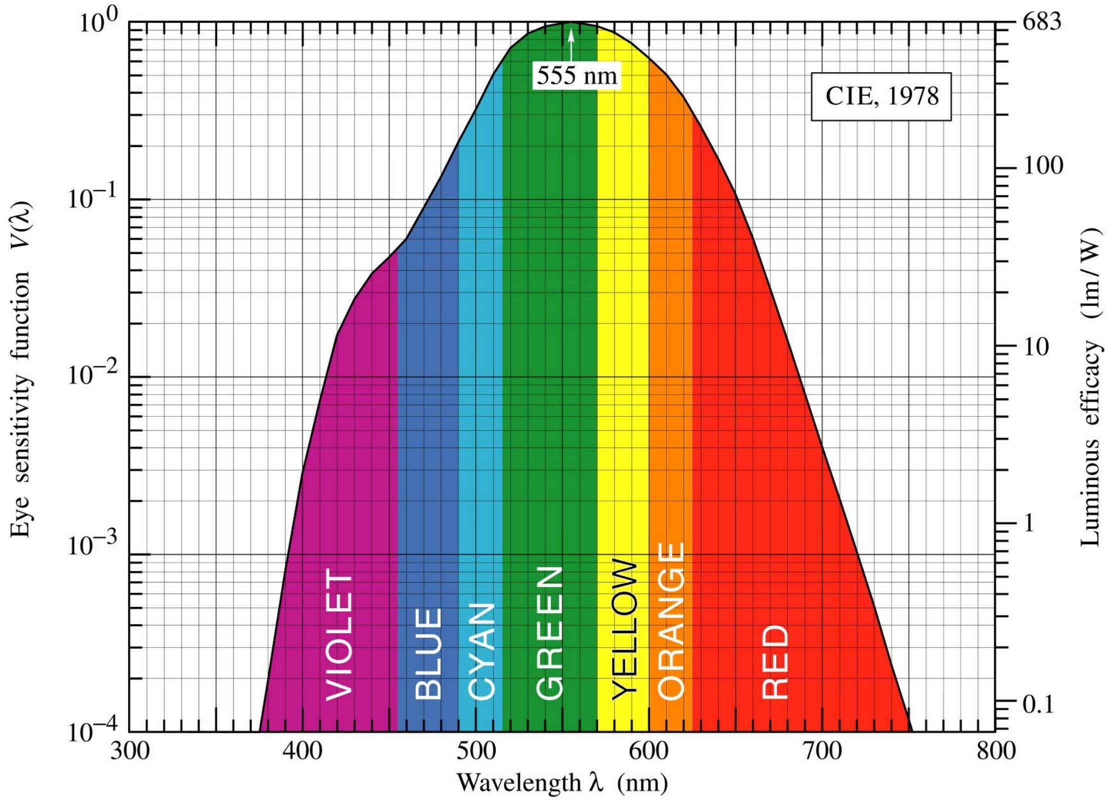


Fig. 11.2. Eye sensitivity function, $V(\lambda)$, (left ordinate) and luminous efficacy, measured in lumens per Watt of optical power (right ordinate). The eye sensitivity is greatest at 555 nm. Also given is a polynomial approximation for $V(\lambda)$ (after 1978 CIE data).

Eye sensitivity function, luminous efficacy, and luminous efficiency

The conversion between radiometric units and photometric units is provided by the ***eye sensitivity function***, $V(\lambda)$, shown in ***Fig***. 11.2. The eye has its peak sensitivity in the green at 555 nm where the eye sensitivity function has a value of unity, *i. e.* V(555 nm) = 1. Numerical values of $V(\lambda)$ are tabulated in Appendix 11.1. For wavelengths ranging from 390 to 720 nm, the eye sensitivity function is greater than 10^{-3}. Although the human eye is sensitive to light with wavelengths < 390 nm and > 720 nm, the sensitivity at these wavelengths is extremely low. The relationship between color and wavelength is given in ***Table*** 11.2.

Color	***Wavelength***
Ultraviolet	< 390 nm
Violet	390 to 455 nm
Blue	455 to 490 nm
Cyan	490 to 515 nm
Green	515 to 570 nm

Color	***Wavelength***
Yellow	570 to 600 nm
Amber	590 to 600 nm
Orange	600 to 625 nm
Red	625 to 720 nm
Infrared	> 720 nm

Table 11.2. Colors and associated typical wavelength ranges.

The ***luminous flux***, Φ_{lum}, is obtained from the radiometric light power using the equation

$$\boxed{\Phi_{\text{lum}} = 683 \frac{\text{lm}}{\text{W}} \int_\lambda V(\lambda)\, P(\lambda)\, \mathrm{d}\lambda} \tag{11.1}$$

where $P(\lambda)$ is the optical power spectrum, *i. e.* the light power emitted per unit wavelength, and the prefactor 683 lm / W is a normalization factor. The optical power emitted by a light source is then given by

$$P = \int_\lambda P(\lambda)\, \mathrm{d}\lambda \tag{11.2}$$

The ***luminous efficacy of optical radiation***, measured in units of lumens per Watt of optical power, is the conversion efficiency from optical power to luminous flux. The luminous efficacy is defined as

$$\boxed{\text{Luminous efficacy} = \Phi_{\text{lum}}/P = \left(683 \frac{\text{lm}}{\text{W}} \int_\lambda V(\lambda)\, P(\lambda)\, \mathrm{d}\lambda \right) \Big/ \left(\int_\lambda P(\lambda)\, \mathrm{d}\lambda \right)} \tag{11.3}$$

Amber light has a higher luminous efficacy than red light. For strictly monochromatic light sources ($\Delta\lambda \to 0$), the luminous efficacy is equal to the eye sensitivity function $V(\lambda)$ multiplied by 683 lm / W. However, for multicolor light sources and especially for white light sources, the luminous efficacy needs to be calculated by integration over all wavelengths.

The ***luminous efficiency of a light source***, also measured in units of lm / W, is the luminous flux of the light source divided by the electrical input power.

$$\boxed{\text{Luminous efficiency} = \Phi_{\text{lum}} \,/\, (I V)} \tag{11.4}$$

where the product ($I\,V$) is the electrical input power of the device (in the lighting community, luminous efficiency is often referred to as *luminous efficacy of the source*).

Inspection of Eqs. (11.3) and (11.4) yields that the luminous efficiency is the product of luminous efficacy and electrical-to-optical power conversion efficiency. The luminous efficiency of common light sources is given in ***Table*** 11.3.

Light source		Luminous efficiency
Edison's first light bulb	(a)	1.4 lm / W
Tungsten filament light bulbs	(a)	15 to 20 lm / W
Quartz halogen light bulbs	(a)	20 to 25 lm / W
Fluorescent light tubes and compact bulbs	(b)	50 to 80 lm / W
Mercury vapor light bulbs	(c)	50 to 60 lm / W
Metal halide light bulbs	(c)	80 to 125 lm / W
High-pressure sodium vapor light bulbs	(c)	100 to 140 lm / W

Table 11.3. Luminous efficiency of different light sources. (a) Incandescent sources. (b) Fluorescent sources. (c) High intensity discharge (HID) sources.

The luminous efficiency is a highly relevant figure of merit for visible LEDs. It is a measure of the perceived light power normalized to the electrical power expended to operate the LED. For light sources with a perfect electrical-power-to-optical-power conversion, the luminous efficiency is equal to the luminous efficacy.

Exercise. ***Luminous efficacy and luminous efficiency of LEDs***. Consider a red and an amber LED emitting at 625 nm and 590 nm, respectively. For simplicity assume that the emission spectra are monochromatic ($\Delta\lambda \rightarrow 0$). What is the luminous efficacy of the two light sources? Calculate the luminous efficiency of the LEDs, assuming that the red and amber LEDs have an external quantum efficiency of 50 %. Assume that the LEDs are driven by a current of 10 mA and that the LED voltages are given by $V = E_g / e = h\nu / e$.

Assume next that the LED spectra are thermally broadened and have a gaussian lineshape with a linewidth of 1.8 kT. Calculate again the luminous efficacy and luminous efficiency of the two light sources. How accurate are the results obtained with the approximation of monochromaticity?

All LEDs have a certain ***radiation pattern*** or ***far-field pattern***. The intensity, measured in W / cm^2, depends on the longitudinal and azimuth angle and the distance from the LED. The total optical power emitted by the LED is obtained by integration over the area of a sphere.

$$P = \int_A \int_\lambda I(\lambda)\, \mathrm{d}\lambda\, \mathrm{d}A \qquad (11.5)$$

where $I(\lambda)$ is the ***light intensity*** (measured in W per nm per cm^2) and A is the surface area of the sphere. Integration is carried out over the entire surface area of the sphere.

Table 11.4 summarizes frequently used figures of merit for light-emitting diodes.

Figure of merit	***Explanation***	***Unit***
Luminous efficacy	Luminous flux per optical unit power	lm / W
Luminous flux efficiency	Luminous flux per input electrical unit power	lm / W
Luminous intensity efficiency	Luminous flux per sr per input electrical unit power	cd / W
Power efficiency	Optical output power per input electrical unit power	%
Internal quantum efficiency	Photons emitted in active region per electron injected	%
External quantum efficiency	Photons emitted from LED per electron injected	%
Extraction efficiency	Escape probability of photons emitted in active region	%

Table 11.4. Summary of photometric, radiometric, and quantum performance measures for LEDs.

Color matching functions and chromaticity diagram

Light causes different levels of excitation of the red, green, and blue cones in the eye. However, the sensation of color and luminous flux of a particular color varies slightly among different individuals. Furthermore the sensation of color is a subjective quantity that cannot be measured objectively. For these reasons, *The International Commission for Illumination* (*Commission Internationale de l'Eclairage*, CIE) has *standardized the measurement of color* using ***color matching functions*** and the ***chromaticity diagram***. Note that neither, the color matching functions nor the chromaticity diagram, are unique. In fact there have been several different versions of the color matching functions and chromaticity diagram.

The CIE color matching functions are shown in ***Fig***. 11.3. The values of the three color matching functions are tabulated in ***Appendix*** 11.1. The three color matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ approximately correspond to the eye sensitivity curves of the red, green, and blue cones, respectively.

The perception of colored light can be analyzed in terms of the degree the light stimulates the three types of cones. The degree of stimulation allows one to characterize *all* possible colors.

The green color matching function has been chosen in such a way that its numerical value is identical to the eye sensitivity function, *i. e.*

$$\boxed{\bar{y}(\lambda) \;=\; V(\lambda)} \tag{11.6}$$

Note that $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, and $V(\lambda)$ are dimensionless quantities.

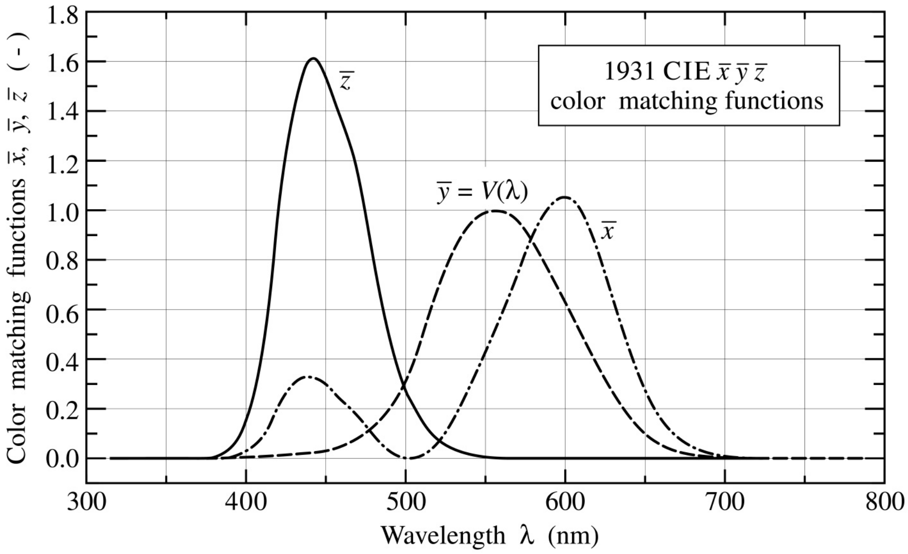


Fig. 11.3. 1931 CIE XYZ color matching functions. The $\bar{y}$ color matching function is identical to the eye sensitivity function $V(\lambda)$.

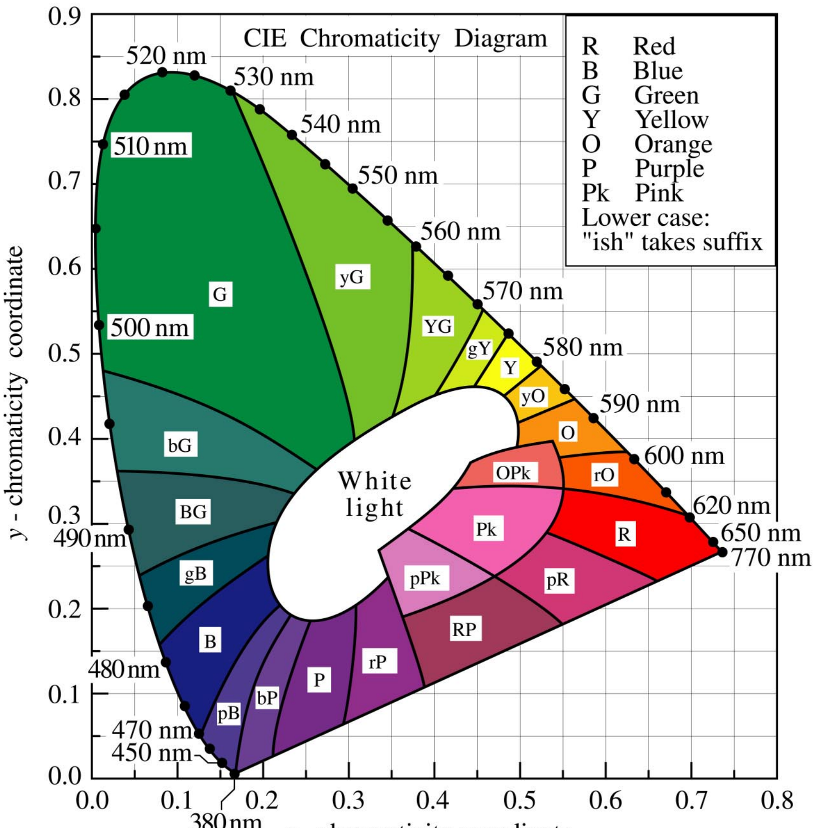


Fig. 11.4. CIE chromaticity diagram. Monochromatic colors are located on the perimeter and white light is located in the center of the diagram (adopted from Gage *et al.*, 1977).

The degree of stimulation of the three types of cones is given by

$$X = \int_{\lambda} \bar{x}(\lambda)\; P(\lambda)\; \mathrm{d}\lambda \qquad (11.7)$$

$$Y = \int_\lambda \bar{y}(\lambda)\ P(\lambda)\ \mathrm{d}\lambda \tag{11.8}$$

$$Z = \int_\lambda \bar{z}(\lambda)\ P(\lambda)\ \mathrm{d}\lambda \tag{11.9}$$

where X, Y, and Z are the ***tristimulus values*** that indicate the relative stimulation of each of the three cones. Equations (11.7) to (11.9) suggest that the unit of the tristimulus values is Watt. A prefactor in front of the integral can be included so that the tristimulus values become dimensionless. If only *ratios* of tristimulus values are employed, as below, the prefactors and units cancel and thus become irrelevant.

The ***chromaticity coordinates*** x and y are calculated from the tristimulus values according to

$$x = \frac{X}{X + Y + Z} \tag{11.10}$$

$$y = \frac{Y}{X + Y + Z}\ . \tag{11.11}$$

Thus the value of a chromaticity coordinate is the stimulation of one particular cone normalized to the entire stimulation ($X + Y + Z$). The value of the z chromaticity coordinate is calculated analogously, that is

$$z = \frac{Z}{X + Y + Z} = 1 - x - y \tag{11.12}$$

Note that the z chromaticity value can be obtained from x and y, so that there is no new information in the z chromaticity coordinate. Therefore, the z coordinate is redundant and, for this reason, does not need to be used.

The chromaticity diagram is shown in ***Fig***. 11.4. Reddish and greenish colors are found for large values of x and y, respectively. Bluish colors are found for large values of z, which is, according to Eq. (11.12), for low values of x and y, or near the origin of the chromaticity diagram.

Monochromatic or pure colors are found on the perimeter of the chromaticity diagram. White light is found in the center of the chromaticity diagram. All colors can be characterized in terms of their location in the chromaticity diagram.

There are chromaticity coordinates other than the (x, y) chromaticity coordinates. In 1960, the CIE introduced the (u, v) and in 1976 the (u', v') ***uniform chromaticity coordinates***. These coordinates form the ***uniform chromaticity diagram***. The uniform chromaticity coordinates are calculated form the tristimulus values according to

$$u = \frac{4X}{X + 15Y + 3Z} \qquad v = \frac{6Y}{X + 15Y + 3Z} \qquad \text{(CIE, 1960)} \tag{11.13}$$

and

$$u' = \frac{4X}{X + 15Y + 3Z} \qquad v' = \frac{9Y}{X + 15Y + 3Z}\ . \qquad \text{(CIE, 1976)} \tag{11.14}$$

The motivation for the introduction of the uniform chromaticity coordinates is as follows: The color differences between to different points in the (x, y) chromaticity diagram is spatially very non-uniform, that is, the color may change much more rapidly in one direction, *e. g.* the x-direction, as compared to the other direction, *e. g.* the y-direction.

This deficiency of the (x, y) chromaticity diagram is strongly reduced, although not eliminated, in the (u, v) and (u', v') uniform chromaticity diagrams. As a result the *color difference* between two locations in the uniform chromaticity diagram is directly proportional to the *geometrical distance* between these points.

There are also other methods to characterize colors. In one such scheme, colors are characterized according to ***hue***, ***saturation***, and ***brightness***. The *hue* gives the *color* of a specific light. The hue can be, for example, red or green. The degree of *saturation* indicates the balance between a single monochromatic color (which has high saturation) and white light (which has low saturation). The color sensation also depends on the *brightness* of the light. Generally the perception of colors decreases or gets totally lost at low light intensities (scotopic vision regime). That is, objects loose their colorful appearance at sufficiently low light levels.

Color purity

Monochromatic sources ($\Delta\lambda \to 0$) are located on the perimeter of the chromaticity diagram. However, as the spectral linewidth of a light source gets broader, the color location in the chromaticity diagram moves towards the center of the chromaticity diagram. If the spectral width of a light source becomes comparable to the entire visible range, the light source is *white* and thus located near the center of the chromaticity diagram.

The ***dominant color*** of a test light source is defined as the monochromatic color located on the perimeter of the chromaticity diagram that appears to be closest to the color of the test light source. The dominant color is determined by drawing a straight line between a white reference illuminant through the (x, y) chromaticity coordinate of the test light source to be measured, to the perimeter of the chromaticity diagram. The intersection point is the dominant color of the light source. The procedure to find the dominant wavelength is schematically shown in ***Fig***. 11.5.

The ***color purity*** or ***color saturation*** of a light source is the distance in the chromaticity diagram from the (x, y) color coordinate point to the coordinate of a white reference illuminant normalized by the distance between the white reference illuminant to the dominant color. The color purity is given by

$$\text{Color purity} = \frac{a}{a+b} = \frac{\sqrt{(x - x_{\text{ref}})^2 + (y - y_{\text{ref}})^2}}{\sqrt{(x_{\text{d}} - x_{\text{ref}})^2 + (y_{\text{d}} - y_{\text{ref}})^2}} \tag{11.15}$$

where a and b are shown in ***Fig***. 11.5 and (x, y), $(x_{\text{ref}}, y_{\text{ref}})$, and $(x_{\text{d}}, y_{\text{d}})$ represent the chromaticity coordinates of the light source under test, of the reference illuminant, and of the dominant color, respectively. Generally, the color purity is 100 % for monochromatic light sources ($\Delta\lambda \to 0$) located on the perimeter of the chromaticity diagram and 0 % for white illuminants.

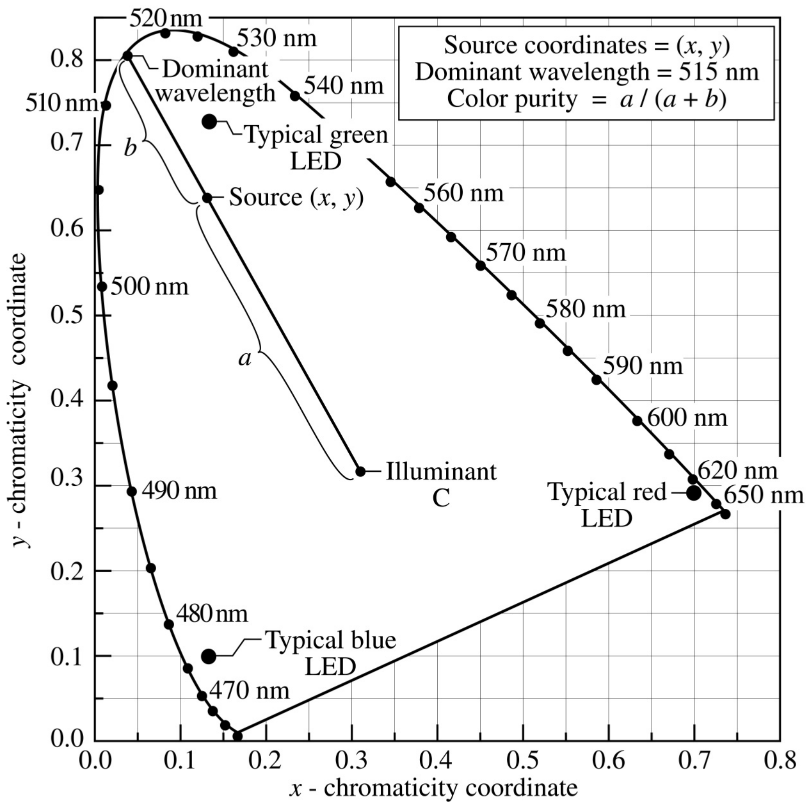


Fig. 11.5. Chromaticity diagram showing the determination of the dominant color and color purity of a light source with chromaticity coordinates (x, y) using the Illuminant C as the white-light reference. Also shown are typical locations of blue, green, and red LEDs.

The color purity is illustrated in ***Fig***. 11.5 as the relative distance of the light source under test from the center of the chromaticity diagram. Also shown in ***Fig***. 11.5 are the typical locations of a GaInN green LED, a AlGaInP red LED, and a GaInN blue LED. The color purity of the red LED is very high, very close to 100 %. The green LED has a lower color purity due to the non-zero spectral width of the LED emission and the strong curvature of the chromaticity diagram in the green region.

LEDs in the chromaticity diagram

Monochromatic light sources ($\Delta\lambda \rightarrow 0$) are located on the perimeter of the chromaticity diagram. Light emission from LEDs is monochromatic (single color) to the eye but LEDs are not monochromatic in the strict physical sense since LEDs have a spectral linewidth of about 1.8 kT. Due to the finite spectral linewidth of LEDs, they are not located on the very perimeter of the chromaticity diagram but are located *close* to the perimeter. When a source emits light distributed over a range of wavelengths, then the location moves towards the center of the diagram.

The location of different LEDs on the chromaticity diagram is shown in ***Fig***. 11.6. Inspection of the figure reveals that the location of red and blue LEDs is on the perimeter of the chromaticity diagram. However, blue-green and green LEDs are located off the perimeter closer to the center of the diagram due to the finite linewidth of the emission spectrum and the strong curvature of the chromaticity diagram in the green wavelength range.

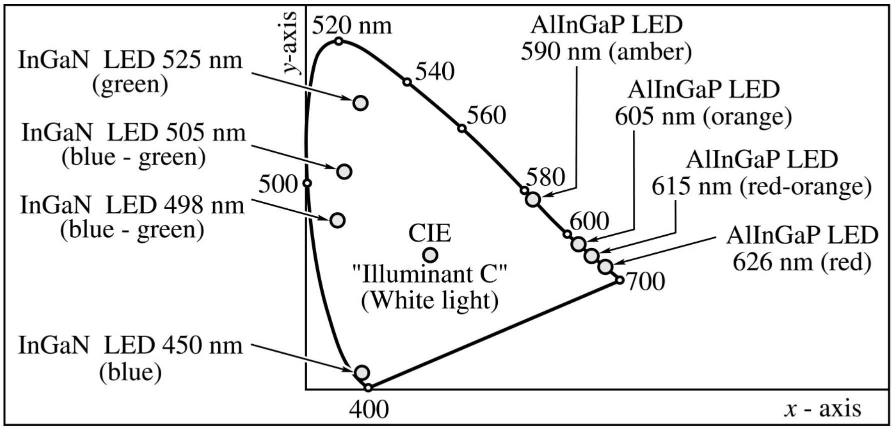


Fig. 11.6. Location of LED light emission on the chromaticity diagram (adopted from Schubert and Miller, 1999).

White illuminants and color temperature

White light usually has a broad spectrum extending over the entire visible range. The ideal model for white light is the sunlight. The sun's optical spectrum is shown in ***Fig***. 11.7 including the spectrum incident above the earth's atmosphere, and at sea level. The spectrum of sunlight extends over the entire visible region. However, the sun's spectrum depends on the time of day, the season, the altitude, the weather, and other factors.

It is desirable to define an independent standard for white light. The ***black-body radiation spectrum*** is used as one such standard. The black-body spectrum is characterized by only one parameter, the temperature of the body. The black-body spectrum was first derived by Max Planck (1900) and is given by

$$\boxed{I(\lambda) = \frac{4\pi\hbar c^2}{\lambda^5\left(\exp\left(\frac{2\pi\hbar c}{\lambda kT}\right) - 1\right)}} \tag{11.16}$$

The maximum intensity of the radiation emanating from a black body with temperature T occurs at the wavelength given by ***Wien's law***

$$\lambda_{max} = \frac{2880\ \mu\text{m K}}{T} \tag{11.17}$$

At "low" black-body temperatures, *e. g*. 2,000 K, the radiation occurs mostly in the infrared. As the temperature increases, the maximum of the radiation moves into the visible wavelength range.

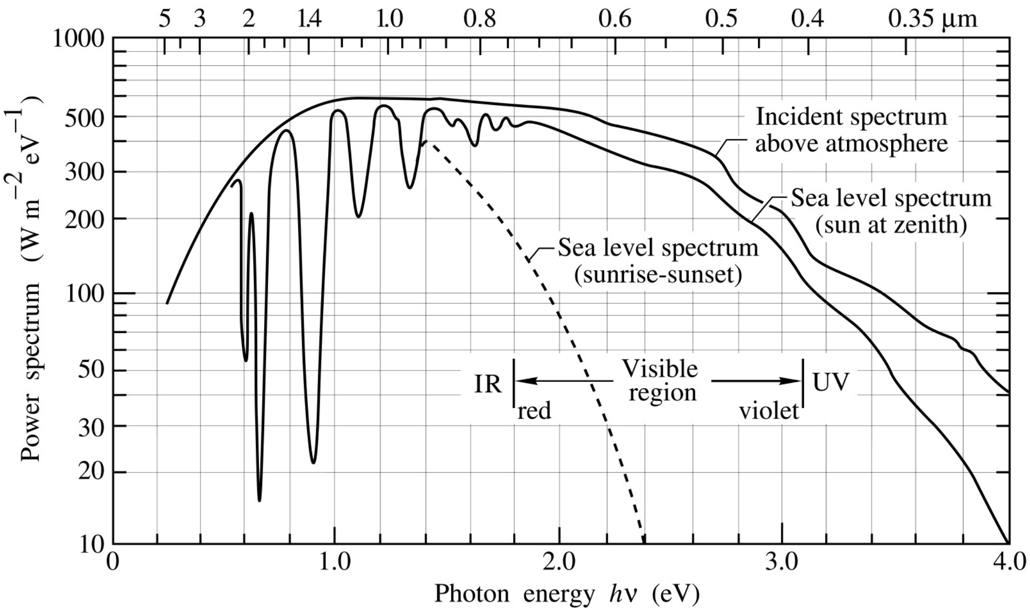


Fig. 11.7. Power spectrum of solar radiation versus photon energy and wavelength for different conditions (adopted from Jackson, 1975).

The location of the black-body radiation in the chromaticity diagram is shown in ***Fig***. 11.8. As the temperature of the black body increases, the chromaticity location moves from the red wavelength range towards the center of the diagram. Typical black-body temperatures in the white region of the chromaticity diagram range between 2,500 and 10,000 K. Also shown in ***Fig***. 11.8 are the locations of several illuminants standardized by the CIE. These standard illuminants include Illuminant A, B, C, and D_{65}.

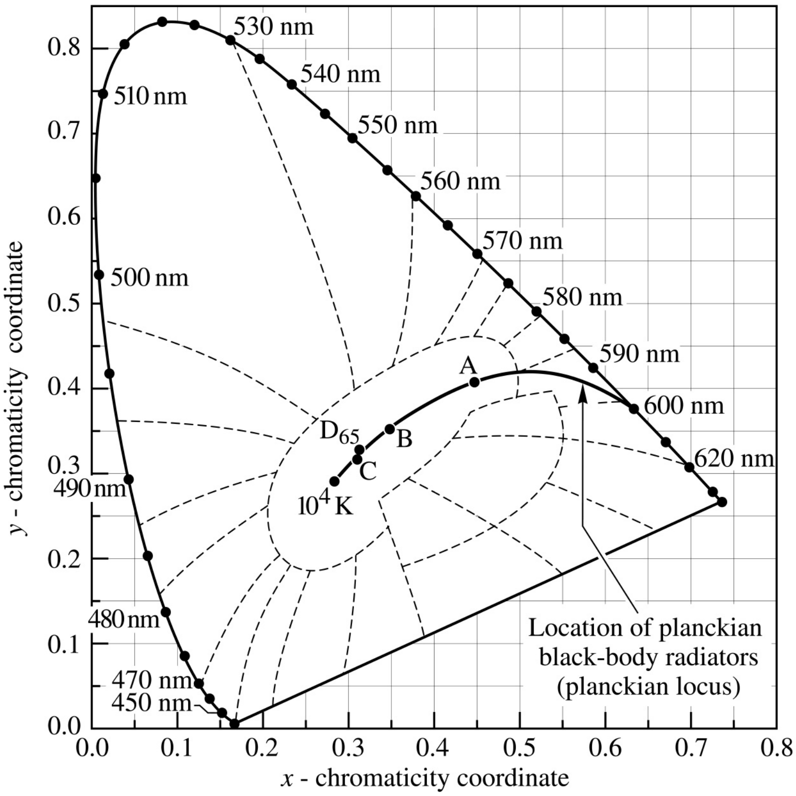
The ***color temperature*** (CT) of a white light source, given in units of Kelvin, is the temperature of a planckian black-body radiator that has the same chromaticity location as the white light source considered.

If the color of a white light source does not fall on the planckian locus, the ***correlated color temperature*** (CCT), also given in units of Kelvin, is used. The correlated color temperature of a white light source is defined as the temperature of a planckian black-body radiator, whose color is closest to the color of the white light source.

The correlated color temperature of a light source is determined as follows. On the (u, v) uniform chromaticity diagram, the point on the planckian locus that is *closest* to the chromaticity location of the light source, is determined. The correlated color temperature is the temperature of the planckian black-body radiator at that point. The determination of the correlated color temperature was discussed in CIE publication No. 17.4 (1987) and by Robertson (1968).

The chromaticity locations of incandescent light sources are very close to, although not exactly on the planckian locus (Ohno, 2000). For such incandescent light sources, the color temperature can be specified. Standard incandescents have color temperatures ranging from 2,000 K to 2,900 K. The common warm incandescent light source has a color temperature of 2,800 K. Quartz halogen incandescent lamps have a color temperature ranging from 2,800 K to 3,200 K (Ohno, 1997).

Other light sources, such as metal halide light sources, are further removed from the planckian locus. For such light sources, the correlated color temperature should be used. Bluish white lights have a correlated color temperature of about 8,000 K.



Illuminant A
$(x, y) = (0.4476, 0.4074)$
(Incandescent source, $T = 2856$ K)

Illuminant B
$(x, y) = (0.3484, 0.3516)$
(Direct sunlight, $T = 4870$ K)

Illuminant C
$(x, y) = (0.3101, 0.3162)$
(Overcast source, $T = 6770$ K)

Illuminant D_{65}
$(x, y) = (0.3128, 0.3292)$
(Daylight, $T = 6500$ K)

Fig. 11.8. Chromaticity diagram showing the standardized white Illuminants A, B, C, and D_{65} and their color temperature (after CIE, 1978).

Note that white light can be created in several different ways. These ways include the creation of white light by ***broad-band*** emission, by the employment of a ***dichromatic*** source (a source emitting two complementary colors), or a ***trichromatic*** source (a source emitting three colors).

Additive color mixing

The *combination* or *additive mixing* of two or more light sources is employed in a number of applications. In LED displays, three different types of LEDs, usually emitting in the red, green, and blue, are used. The three colors are mixed so that the observer sees a mixture of the three colors. Another example of color mixing are white-light emitters based on two or three complementary colors.

We next determine the chromaticity coordinates of a light source emitting three discrete emission bands. Assume that each emission band is much narrower than any of the three color matching functions. Consider three sources with spectral power density $P_1(\lambda)$, $P_2(\lambda)$, and $P_3(\lambda)$ with peak wavelengths λ_1, λ_2, λ_3. Assume further that the three light sources have the chromaticity coordinates (x_1, y_1), (x_2, y_2), and (x_3, y_3). If the spectral linewidth of the three sources is much narrower than the color matching function, then the integrals of Eqs. (11.7) to (11.9) can be rewritten as

$$X = \bar{x}(\lambda_1)\, P_1 + \bar{x}(\lambda_2)\, P_2 + \bar{x}(\lambda_3)\, P_3 \tag{11.18}$$

$$Y = \bar{y}(\lambda_1)\, P_1 + \bar{y}(\lambda_2)\, P_2 + \bar{y}(\lambda_3)\, P_3 \tag{11.19}$$

$$Z = \bar{z}(\lambda_1)\, P_1 + \bar{z}(\lambda_2)\, P_2 + \bar{z}(\lambda_3)\, P_3 \tag{11.20}$$

where P_1, P_2, and P_3 are the optical powers emitted by the three sources. Using the abbreviations

$$L_1 = \bar{x}(\lambda_1)\, P_1 + \bar{y}(\lambda_1)\, P_1 + \bar{z}(\lambda_1)\, P_1 \tag{11.21}$$

$$L_2 = \bar{x}(\lambda_2)\, P_2 + \bar{y}(\lambda_2)\, P_2 + \bar{z}(\lambda_2)\, P_2 \tag{11.22}$$

$$L_3 = \bar{x}(\lambda_3)\, P_3 + \bar{y}(\lambda_3)\, P_3 + \bar{z}(\lambda_3)\, P_3 \tag{11.23}$$

the chromaticity coordinates of the combined light are calculated according to Eqs. (11.10) and (11.11) and are given by

$$x = \frac{x_1 L_1 + x_2 L_2 + x_3 L_3}{L_1 + L_2 + L_3} \tag{11.24}$$

$$y = \frac{y_1 L_1 + y_2 L_2 + y_3 L_3}{L_1 + L_2 + L_3} \tag{11.25}$$

Thus the chromaticity coordinate of the multi-component light is a linear combination of the individual chromaticity coordinates weighted by the L_i factors.

The principle of color mixing is shown in ***Fig***. 11.9. The figure shows the mixing of *two* colors with chromaticity coordinates (x_1, y_1) and (x_2, y_2). For the case of two colors, $L_3 = P_3 = 0$. The mixed color will be located on the straight line connecting the chromaticity coordinates of the two light sources. Thus *any* color (including white) located between the two chromaticity points can be created by mixing the two colors.

Figure 11.9 also shows the mixing of *three* colors, located in the red, green, and blue region of the chromaticity diagram. The three chromaticity points are typical points for red, green, and blue LEDs. The three points are connected by a dashed line. The region within the dashed line, often referred to as the ***color gamut***, represents all colors that can be created by mixing primary colors, *e. g.* red, green, and blue. The ability to create a great variety of colors is an important quality for displays. It is desirable that the area provided by the three light sources is as large as possible to create brilliant displays.

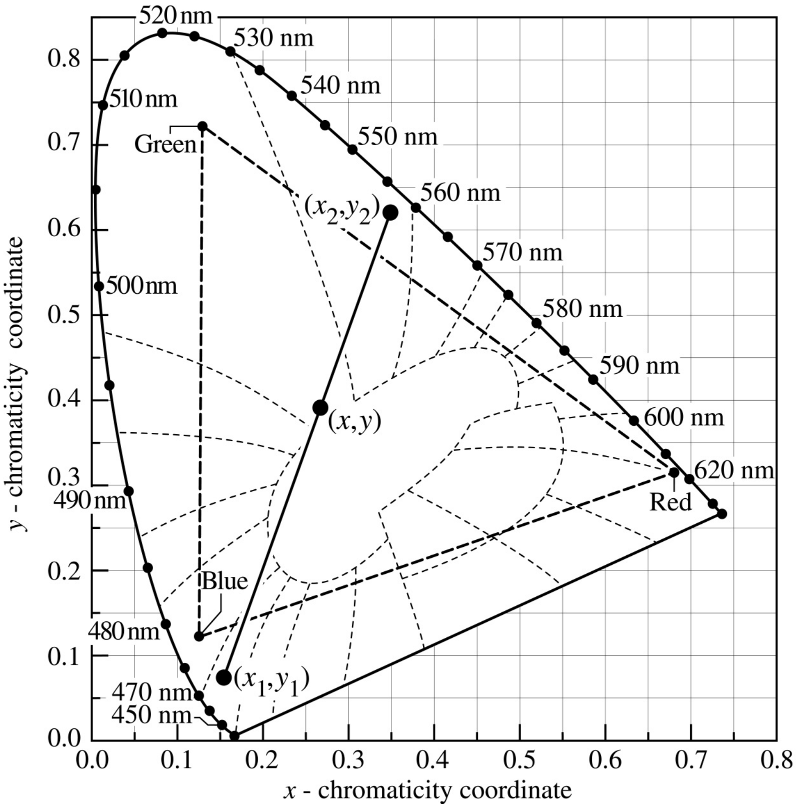


Fig. 11.9. Principle of color mixing illustrated with two light sources with chromaticity coordinates (x_1, y_1) and (x_2, y_2). The resulting color has the coordinates (x, y). Also shown is the triangular area of the chromaticity diagram (color gamut) accessible by additive mixing of a red, green, and blue LED.

The color gamut represents the entire range of colors that can be created from a set of primary colors. Color gamuts are polygons positioned in the chromaticity diagram. For the case of *three* primary colors, the color gamut is a *triangle*, as shown in ***Fig***. 11.9. All colors created by additive mixtures of the vertex points (primary colors) of a gamut, are necessarily located inside the gamut.

The insight now gained on color mixing allows one to understand the location of different LEDs in the chromaticity diagram. The perimeter of the chromaticity diagram in the red spectral region is approximately a *straight line*, so that red LEDs, despite their thermal broadening, are located on the perimeter. In contrast, the perimeter of the chromaticity diagram is strongly curved in the green region, so that green LEDs due to their spectral broadening, are displaced from the perimeter towards the center of the chromaticity diagram.

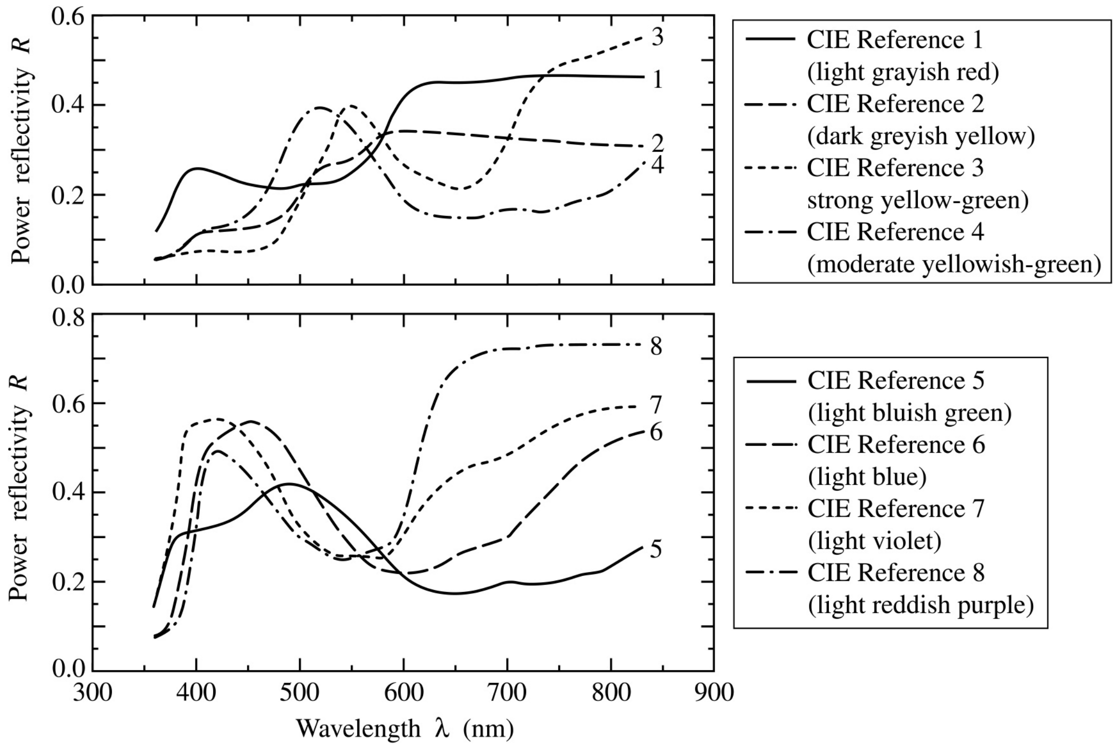


Fig. 11.10. Reflectivity curves of eight sample objects used for the calculation of the general color rendering index (CRI) of light sources used for illumination purposes (after CIE data, 1978).

Color rendering index

Another important quality of white light emitters is the ***color-rendering index*** or ***CRI***. It is a measure of the ability of an *illuminant*, *i. e.* illumination source, to render all the colors of an object illuminated with the light source. The color rendering characteristics of a white light source are relevant for light sources used for illumination purposes, *i. e.* when rendering of the colors of an object is important. The color-rendering index is irrelevant for white light sources used in indicator lamp and signage applications.

The color rendering ability of a ***test light source*** is evaluated by comparing it with a ***reference light source***. The reference light source for calculation of the CRI is usually a planckian black-body radiator with the same correlated color temperature, or one of the standardized illuminants. The CIE recommends that the reference light source be a planckian black-body radiator using the correlated color temperature as the black-body temperature (CIE, 1995). By definition, the reference light source has ideal color rendering properties and its color rendering index is CRI = 100. Illuminants other than the chosen reference light source have a color rendering index lower than 100. Note that the calculation of the CRI depends on the choice of the reference light source. The selection of the reference light source is therefore critical when comparing the CRIs of different light sources.

Incandescent quartz halogen lamps have one of the best color rendering properties of artificial light sources. Such lamps are used in locations where color rendering is of prime importance such as in museums, art galleries, and clothing shops. The drawback of quartz halogen lamps is high power consumption.

The CRI calculation has been discussed in detail by Wyszecki and Stiles (1982). The 1976 CIE general CRI is calculated according to

$$CRI_{\text{general}} = \frac{1}{8} \sum_{i=1}^{8} CRI_i \tag{11.26}$$

where the CRI_i are the ***special color rendering indices*** for eight sample objects. The reflectivity curves of these eight sample objects are shown in ***Fig***. 11.10. The special color rendering indices are given by

$$CRI_i = 100 - 4.6\,\Delta E_i^* \tag{11.27}$$

where ΔE_i* is the difference in color that occurs when a sample object is illuminated with the reference illumination source and the test illumination source. The special color-rendering index is calculated in such a way that it is 100, if there is no color difference when the sample object is illuminated with the reference source and the test source. At the time Eq. (11.27) was established, the pre-factor 4.6 had been chosen in such a way that the general CRI become equal to about 60 when a "standard warm white" fluorescent lamp was used as a test source and an incandescent lamp as a reference source. Current fluorescent light sources have higher CRIs, typically in the range 60 to 85 (Kendall and Scholand, 2001).

The color difference is calculated according to

$$\Delta E_i^* = \sqrt{(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2} \tag{11.28}$$

where

$$\Delta L^* = L^*_{\text{test}} - L^*_{\text{ref}} = \left[116 \left(\frac{Y_{\text{test,reflected}}}{Y_{\text{ref,direct}}} \right)^{1/3} - 16 \right] - \left[116 \left(\frac{Y_{\text{ref,reflected}}}{Y_{\text{ref,direct}}} \right)^{1/3} - 16 \right] \tag{11.29}$$

$$\Delta u^* = u^*_{\text{test}} - u^*_{\text{ref}} = 13\, L^*_{\text{test}} \left(u_{\text{test,reflected}} - u_{\text{ref,direct}} \right) - 13\, L^*_{\text{ref}} \left(u_{\text{ref,reflected}} - u_{\text{ref,direct}} \right) \tag{11.30}$$

$$\Delta v^* = v^*_{\text{test}} - v^*_{\text{ref}} = 13\, L^*_{\text{test}} \left(v_{\text{test,reflected}} - v_{\text{ref,direct}} \right) - 13\, L^*_{\text{ref}} \left(v_{\text{ref,reflected}} - v_{\text{ref,direct}} \right) \tag{11.31}$$

and

$$u = \frac{4\,X}{X + 15\,Y + 3\,Z} \qquad \text{and} \qquad v = \frac{9\,Y}{X + 15\,Y + 3\,Z}\,. \tag{11.32}$$

Note that u and v are calculated from the tristimulus values of the reference source spectrum (Subscript "ref,direct"), from the reference source spectrum reflected off of the sample objects (Subscript "ref,reflected"), and from the test light source spectrum reflected off of the sample objects (Subscript "test,reflected").

The calculation of the color rendering index and the choice of the numerical prefactors in Eqs. (11.27) to (11.32) is somewhat arbitrary. These prefactors have been determined in experiments with human subjects. Evidence exists that the current prefactors are not optimum (Wyszecki and Stiles, 1982).

Light source	***Color rendering index***	
Sunlight	100	(a)
Quartz halogen W filament incandescent light	100	(b)
W filament incandescent light	100	(b)
Fluorescent light	60 to 85	(b)
Trichromatic white light LED	60 to 85	(b, c)
Phosphor-based white LED	55 to 85	(b, c)
Broadened dichromatic white light LED	10 to 60	(b, c)
Hg vapor light coated with phosphor	50	(b)
Hg vapor light	33	(b)
Low and high-pressure Na vapor light	10 and 22	(b)
Green monochromatic light	– 50	(c)

Table 11.5. Color rendering indices (CRIs) of different light sources. (a) Using sunlight as reference source. (b) Using incandescent light with either the same correlated color temperature as reference light source. (c) Using Illuminant D_{65} as reference light source (some data after Kendall and Scholand, 2001).

The color rendering indices of common light sources are summarized in ***Table*** 11.5. The table also includes the color-rendering index of several types of LED sources including dichromatic white LEDs, trichromatic white LEDs, and phosphor-based white LEDs.

Exercise. ***Color rendering***. The color of objects strongly depends on the light source illuminating the objects. Some light sources render the natural colors of an object (true color rendering) and some light sources do not (false color rendering).
(a) What is the color of a yellow banana when illuminated with a red LED?
(b) What is the color of a green banana when illuminated with a yellow LED?
(c) Could it be advantageous for a grocer to illuminate meat with red LEDs, bananas with yellow LEDs, and oranges with orange LEDs?
(d) Is it possible for two objects of different colors to appear to have the same color under certain illumination conditions?
(e) Why are low-pressure Na vapor lights used despite their low color-rendering index?
(f) What would be the advantage and disadvantage of using green LEDs for illumination?
Solution: (a) Red. (b) Yellow. (c) Yes. (d) Yes. (e) Because of high luminous efficiency. (f) High luminous efficacy but low color rendering properties.

Appendix 11.1

Tabulated values of the color matching functions and eye sensitivity function (after CIE, 1978). The color matching functions given here are officially called the Judd-Vos-modified CIE 2-degree color matching functions.

λ	$\bar{x}(\lambda)$ ***red***	$\bar{y}(\lambda) = V(\lambda)$ ***green***	$\bar{z}(\lambda)$ ***blue***
380 nm	2.6899E-3	2.0000E-4	1.2260E-2
385 nm	5.3105E-3	3.9556E-4	2.4222E-2
390 nm	1.0781E-2	8.0000E-4	4.9250E-2
395 nm	2.0792E-2	1.5457E-3	9.5135E-2
400 nm	3.7981E-2	2.8000E-3	1.7409E-1
405 nm	6.3157E-2	4.6562E-3	2.9013E-1
410 nm	9.9941E-2	7.4000E-3	4.6053E-1
415 nm	1.5824E-1	1.1779E-2	7.3166E-1
420 nm	2.2948E-1	1.7500E-2	1.0658
425 nm	2.8108E-1	2.2678E-2	1.3146
430 nm	3.1095E-1	2.7300E-2	1.4672
435 nm	3.3072E-1	3.2584E-2	1.5796
440 nm	3.3336E-1	3.7900E-2	1.6166
445 nm	3.1672E-1	4.2391E-2	1.5682
450 nm	2.8882E-1	4.6800E-2	1.4717
455 nm	2.5969E-1	5.2122E-2	1.3740
460 nm	2.3276E-1	6.0000E-2	1.2917
465 nm	2.0999E-1	7.2942E-2	1.2356
470 nm	1.7476E-1	9.0980E-2	1.1138
475 nm	1.3287E-1	1.1284E-1	9.4220E-1
480 nm	9.1944E-2	1.3902E-1	7.5596E-1
485 nm	5.6985E-2	1.6987E-1	5.8640E-1
490 nm	3.1731E-2	2.0802E-1	4.4669E-1
495 nm	1.4613E-2	2.5808E-1	3.4116E-1
500 nm	4.8491E-3	3.2300E-1	2.6437E-1
505 nm	2.3215E-3	4.0540E-1	2.0594E-1
510 nm	9.2899E-3	5.0300E-1	1.5445E-1
515 nm	2.9278E-2	6.0811E-1	1.0918E-1
520 nm	6.3791E-2	7.1000E-1	7.6585E-2
525 nm	1.1081E-1	7.9510E-1	5.6227E-2
530 nm	1.6692E-1	8.6200E-1	4.1366E-2
535 nm	2.2768E-1	9.1505E-1	2.9353E-2
540 nm	2.9269E-1	9.5400E-1	2.0042E-2
545 nm	3.6225E-1	9.8004E-1	1.3312E-2
550 nm	4.3635E-1	9.9495E-1	8.7823E-3
555 nm	5.1513E-1	1.0000	5.8573E-3
560 nm	5.9748E-1	9.9500E-1	4.0493E-3
565 nm	6.8121E-1	9.7875E-1	2.9217E-3
570 nm	7.6425E-1	9.5200E-1	2.2771E-3
575 nm	8.4394E-1	9.1558E-1	1.9706E-3
580 nm	9.1635E-1	8.7000E-1	1.8066E-3
585 nm	9.7703E-1	8.1623E-1	1.5449E-3
590 nm	1.0230	7.5700E-1	1.2348E-3
595 nm	1.0513	6.9483E-1	1.1177E-3
600 nm	1.0550	6.3100E-1	9.0564E-4
605 nm	1.0362	5.6654E-1	6.9467E-4
610 nm	9.9239E-1	5.0300E-1	4.2885E-4
615 nm	9.2861E-1	4.4172E-1	3.1817E-4
620 nm	8.4346E-1	3.8100E-1	2.5598E-4
625 nm	7.3983E-1	3.2052E-1	1.5679E-4
630 nm	6.3289E-1	2.6500E-1	9.7694E-5
635 nm	5.3351E-1	2.1702E-1	6.8944E-5
640 nm	4.4062E-1	1.7500E-1	5.1165E-5
645 nm	3.5453E-1	1.3812E-1	3.6016E-5
650 nm	2.7862E-1	1.0700E-1	2.4238E-5
655 nm	2.1485E-1	8.1652E-2	1.6915E-5
660 nm	1.6161E-1	6.1000E-2	1.1906E-5
665 nm	1.1820E-1	4.4327E-2	8.1489E-6
670 nm	8.5753E-2	3.2000E-2	5.6006E-6
675 nm	6.3077E-2	2.3454E-2	3.9544E-6
680 nm	4.5834E-2	1.7000E-2	2.7912E-6
685 nm	3.2057E-2	1.1872E-2	1.9176E-6
690 nm	2.2187E-2	8.2100E-3	1.3135E-6
695 nm	1.5612E-2	5.7723E-3	9.1519E-7
700 nm	1.1098E-2	4.1020E-3	6.4767E-7
705 nm	7.9233E-3	2.9291E-3	4.6352E-7
710 nm	5.6531E-3	2.0910E-3	3.3304E-7
715 nm	4.0039E-3	1.4822E-3	2.3823E-7
720 nm	2.8253E-3	1.0470E-3	1.7026E-7
725 nm	1.9947E-3	7.4015E-4	1.2207E-7
730 nm	1.3994E-3	5.2000E-4	8.7107E-8
735 nm	9.6980E-4	3.6093E-4	6.1455E-8
740 nm	6.6847E-4	2.4920E-4	4.3162E-8
745 nm	4.6141E-4	1.7231E-4	3.0379E-8
750 nm	3.2073E-4	1.2000E-4	2.1554E-8
755 nm	2.2573E-4	8.4620E-5	1.5493E-8
760 nm	1.5973E-4	6.0000E-5	1.1204E-8
765 nm	1.1275E-4	4.2446E-5	8.0873E-9
770 nm	7.9513E-5	3.0000E-5	5.8340E-9
775 nm	5.6087E-5	2.1210E-5	4.2110E-9
780 nm	3.9541E-5	1.4989E-5	3.0383E-9
785 nm	2.7852E-5	1.0584E-5	2.1907E-9
790 nm	1.9597E-5	7.4656E-6	1.5778E-9
795 nm	1.3770E-5	5.2592E-6	1.1348E-9
800 nm	9.6700E-6	3.7028E-6	8.1565E-10
805 nm	6.7918E-6	2.6076E-6	5.8626E-10
810 nm	4.7706E-6	1.8365E-6	4.2138E-10
815 nm	3.3550E-6	1.2950E-6	3.0319E-10
820 nm	2.3534E-6	9.1092E-7	2.1753E-10
825 nm	1.6377E-6	6.3564E-7	1.5476E-10

Appendix 11.2

Tabulated values of the reflectivity data of eight sample objects used for the calculation of the color-rendering index (after CIE, 1978).

λ (nm)	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
380 nm	0.219	0.07	0.065	0.074	0.295	0.151	0.378	0.104
385 nm	0.239	0.079	0.068	0.083	0.306	0.203	0.459	0.129
390 nm	0.252	0.089	0.07	0.093	0.31	0.265	0.524	0.17
395 nm	0.256	0.101	0.072	0.105	0.312	0.339	0.546	0.24
400 nm	0.256	0.111	0.073	0.116	0.313	0.41	0.551	0.319
405 nm	0.254	0.116	0.073	0.121	0.315	0.464	0.555	0.416
410 nm	0.252	0.118	0.074	0.124	0.319	0.492	0.559	0.462
415 nm	0.248	0.12	0.074	0.126	0.322	0.508	0.56	0.482
420 nm	0.244	0.121	0.074	0.128	0.326	0.517	0.561	0.49
425 nm	0.24	0.122	0.073	0.131	0.33	0.524	0.558	0.488
430 nm	0.237	0.122	0.073	0.135	0.334	0.531	0.556	0.482
435 nm	0.232	0.122	0.073	0.139	0.339	0.538	0.551	0.473
440 nm	0.23	0.123	0.073	0.144	0.346	0.544	0.544	0.462
445 nm	0.226	0.124	0.073	0.151	0.352	0.551	0.535	0.45
450 nm	0.225	0.127	0.074	0.161	0.36	0.556	0.522	0.439
455 nm	0.222	0.128	0.075	0.172	0.369	0.556	0.506	0.426
460 nm	0.22	0.131	0.077	0.186	0.381	0.554	0.488	0.413
465 nm	0.218	0.134	0.08	0.205	0.394	0.549	0.469	0.397
470 nm	0.216	0.138	0.085	0.229	0.403	0.541	0.448	0.382
475 nm	0.214	0.143	0.094	0.254	0.41	0.531	0.429	0.366
480 nm	0.214	0.15	0.109	0.281	0.415	0.519	0.408	0.352
485 nm	0.214	0.159	0.126	0.308	0.418	0.504	0.385	0.337
490 nm	0.216	0.174	0.148	0.332	0.419	0.488	0.363	0.325
495 nm	0.218	0.19	0.172	0.352	0.417	0.469	0.341	0.31
500 nm	0.223	0.207	0.198	0.37	0.413	0.45	0.324	0.299
505 nm	0.225	0.225	0.221	0.383	0.409	0.431	0.311	0.289
510 nm	0.226	0.242	0.241	0.39	0.403	0.414	0.301	0.283
515 nm	0.226	0.253	0.26	0.394	0.396	0.395	0.291	0.276
520 nm	0.225	0.26	0.278	0.395	0.389	0.377	0.283	0.27
525 nm	0.225	0.264	0.302	0.392	0.381	0.358	0.273	0.262
530 nm	0.227	0.267	0.339	0.385	0.372	0.341	0.265	0.256
535 nm	0.23	0.269	0.37	0.377	0.363	0.325	0.26	0.251
540 nm	0.236	0.272	0.392	0.367	0.353	0.309	0.257	0.25
545 nm	0.245	0.276	0.399	0.354	0.342	0.293	0.257	0.251
550 nm	0.253	0.282	0.4	0.341	0.331	0.279	0.259	0.254
555 nm	0.262	0.289	0.393	0.327	0.32	0.265	0.26	0.258
560 nm	0.272	0.299	0.38	0.312	0.308	0.253	0.26	0.264
565 nm	0.283	0.309	0.365	0.296	0.296	0.241	0.258	0.269
570 nm	0.298	0.322	0.349	0.28	0.284	0.234	0.256	0.272
575 nm	0.318	0.329	0.332	0.263	0.271	0.227	0.254	0.274
580 nm	0.341	0.335	0.315	0.247	0.26	0.225	0.254	0.278
585 nm	0.367	0.339	0.299	0.229	0.247	0.222	0.259	0.284
590 nm	0.39	0.341	0.285	0.214	0.232	0.221	0.27	0.295
595 nm	0.409	0.341	0.272	0.198	0.22	0.22	0.284	0.316
600 nm	0.424	0.342	0.264	0.185	0.21	0.22	0.302	0.348
605 nm	0.435	0.342	0.257	0.175	0.2	0.22	0.324	0.384
610 nm	0.442	0.342	0.252	0.169	0.194	0.22	0.344	0.434
615 nm	0.448	0.341	0.247	0.164	0.189	0.22	0.362	0.482
620 nm	0.45	0.341	0.241	0.16	0.185	0.223	0.377	0.528
625 nm	0.451	0.339	0.235	0.156	0.183	0.227	0.389	0.568

630 nm	0.451	0.339	0.229	0.154	0.18	0.233	0.4	0.604
635 nm	0.451	0.338	0.224	0.152	0.177	0.239	0.41	0.629
640 nm	0.451	0.338	0.22	0.151	0.176	0.244	0.42	0.648
645 nm	0.451	0.337	0.217	0.149	0.175	0.251	0.429	0.663
650 nm	0.45	0.336	0.216	0.148	0.175	0.258	0.438	0.676
655 nm	0.45	0.335	0.216	0.148	0.175	0.263	0.445	0.685
660 nm	0.451	0.334	0.219	0.148	0.175	0.268	0.452	0.693
665 nm	0.451	0.332	0.224	0.149	0.177	0.273	0.457	0.7
670 nm	0.453	0.332	0.23	0.151	0.18	0.278	0.462	0.705
675 nm	0.454	0.331	0.238	0.154	0.183	0.281	0.466	0.709
680 nm	0.455	0.331	0.251	0.158	0.186	0.283	0.468	0.712
685 nm	0.457	0.33	0.269	0.162	0.189	0.286	0.47	0.715
690 nm	0.458	0.329	0.288	0.165	0.192	0.291	0.473	0.717
695 nm	0.46	0.328	0.312	0.168	0.195	0.296	0.477	0.719
700 nm	0.462	0.328	0.34	0.17	0.199	0.302	0.483	0.721

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